Rapid Calculation of the Green's Function in the Shielded Planar Structures

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***Abstract*— A rapid summation method is proposed for the potential Green's function in shielded planar multilayered structures. Most terms of the Green's function series are converted into the space domain static image series and are calculated efficiently using the Ewald sum technique. Numerical results show the efficiency and accuracy of the proposed method.**

***Index Terms*—Green's function, shielded planar structure.**

I. Introduction

THE integral equation method of moment (IE-MoM) is a widely used method in the analysis of the shielded planar circuits. One of the problems in the practical application of this method is the excessive amount of computation needed to fill the MoM matrix due to the slow convergence of the relevant Green's function. Although the fast Fourier transform (FFT)-based algorithm [1] gives a practical solution for most problems, it entails inaccuracies or inefficiencies in modeling certain kind of circuit geometries due to the uniform rectangular grid restriction [2]. In this case, more flexible discretization schemes are required along with specialized methods for the fast calculation of the impedance matrix elements. Hashemi–Yeganeh [3] proposed an efficient calculation method based on the residue theorem and the contour integration method. In the scheme of Eleftheriades *et al.* [2], the quasistatic asymptotic part of the Green's function series has been extracted and transformed into a rapidly convergent form. Shubair *et al.* [4] converted the periodic Green's function in layered dielectric media into the complex image series and accelerated it with the acceleration scheme for the free-space periodic Green's function.

This letter proposes a new efficient procedure to calculate the Green's function in the shielded planar structure. In the proposed method, only the higher order real terms of the Green's function series are converted into the space domain and are accelerated. Numerical efficiency and simplicity have been achieved through the static image conversion and the subsequent use of the Ewald sum technique [4]. The final form of the Green's function converges very rapidly, and only a small number of terms of the series are sufficient to get accurate result in the numerical calculation.

II. Theory

For the shielded planar structure whose lateral dimension is a, b, and the height c, the scalar potential Green's function can be written as follows:

$$G^V(\vec{r}|\vec{r}') = \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\tilde{G}^{*V}(k_{mn}) \times \sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y')$$
$$k_{xm} = \frac{m\pi}{a},\;\; k_{yn} = \frac{n\pi}{b},\;\; k_{mn} = \sqrt{k_{xm}^2 + k_{yn}^2} \tag{1}$$

where $\tilde{G}^{*V}$ is the spectral domain Green's function in non-shielded planar structure with top and bottom ground planes. On the real $k_\rho = \sqrt{k_x^2 + k_y^2}$ axis, the spectral domain Green's function has complex value only for $k_\rho < k_{i,\max}(k_{i,\max} = Max\{k_i\}, k_i = \omega\sqrt{\mu_i \epsilon_i})$. Therefore, the Green's function series can be divided into two parts: low-order $(k_{mn} < k_{i,\max})$ complex terms and the higher order$(k_{mn} > k_{i,\max})$ real terms. For the real terms, the spectral domain Green's function is approximated with exponential functions as follows:

$$\tilde{G}^{*V}(k_{mn}) = \frac{1}{k_{mn}}\sum_{i=1}^{N_V} C_i^V \exp(s_i^V k_{mn}),\;\; k_{mn} > k_{i,\max}. \tag{2}$$

The above approximation involves a real function defined on a real axis without any singularities (e.g., surface wave poles). Therefore, neither the path deformation nor the singularity extraction [5] is required and all the calculation can be done using only the real arithmetic. Therefore, the image conversion process becomes simple and efficient. The exponential approximation used the generalized pencil-of-function [6] method.

The approximation result in (2) is substituted for the real terms in the Green's function series, and the approximation is extrapolated into the range of $k_{mn} < k_{i,\max}$. The extrapolated terms are balanced out by subtracting them from the complex terms

$$\begin{aligned} G^V(\vec{r}|\vec{r}') =& \frac{4}{ab}\sum_{m=1}^{(k_{mn}}\sum_{n=1}^{<k_{i,\max})}\left\{\tilde{G}^{*V}(k_{mn}) - \frac{1}{k_{mn}}\sum_{i=1}^{N_V} C_i^V \exp(s_i^V k_{mn})\right\} \\ &\times \sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y') \\ &+ \sum_{i=1}^{N_V} C_i^V \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\exp(s_i^V k_{mn})}{k_{mn}} \\ &\times \sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y'). \end{aligned} \tag{3}$$

Each term in the second series can be converted into the space-domain static image series using the Poisson sum trans-

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formation [4]

$$\begin{aligned}&\frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\exp(s_i^V k_{mn})}{k_{mn}}\\&\cdot\sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y')\\&=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left\{\frac{1}{R_{mn,i1}^V}-\frac{1}{R_{mn,i2}^V}\right.\\&\qquad\left.-\frac{1}{R_{mn,i3}^V}+\frac{1}{R_{mn,i4}^V}\right\}\\R_{mn,ik}^V&=\sqrt{(X+2ma)^2+(Y+2nb)^2+(s_i^V)^2}\\&(X=x-x',k=1,2,X=x+x',k=3,4,\\&Y=y-y',k=1,3,Y=y+y',k=2,4).\end{aligned}\tag{4}$$

Each of the four terms in the image series represents the potential function due to the two-dimensional (2-D) periodic charge distribution. Applying the Ewald sum technique [7] to each term in (4) and merging them together yields the following result:

$$\begin{aligned}&\frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\epsilon_{0m}\epsilon_{0n}\frac{\exp(s_i^V k_{mn})}{k_{mn}}\\&\times\sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y')\\&=G_{i1}^V+G_{i2}^V,\\G_{i1}^V&=\frac{2}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{k_{mn}}\{\exp(s_i^V k_{mn})\mathrm{erfc}(k_{mn}/2E+s_i^V E)\\&+\exp(-s_i^V k_{mn})\mathrm{erfc}(k_{mn}/2E-s_i^V E)\}\\&\times\sin(k_{xm}x)\sin(k_{yn}y)\sin(k_{xm}x')\sin(k_{yn}y')\\G_{i2}^V&=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left\{\frac{\mathrm{erfc}(R_{mn,i1}^V E)}{R_{mn,i1}^V}-\frac{\mathrm{erfc}(R_{mn,i2}^V E)}{R_{mn,i2}^V}\right.\\&\qquad\left.-\frac{\mathrm{erfc}(R_{mn,i3}^V E)}{R_{mn,i3}^V}+\frac{\mathrm{erfc}(R_{mn,i4}^V E)}{R_{mn,i4}^V}\right\}.\end{aligned}\tag{5}$$

The details of the Ewalt sum technique can be found in [7], according to which the parameter E can be set to $\pi/2\sqrt{ab}$. Since $\mathrm{erfc}(\mathrm{x})$ converges at the rate $\exp(-x^2)/x$, the two series in (5) are rapidly convergent.

III. Numerical Result

Fig. 1 shows the structure of a shielded microstrip low-pass filter. The thickness and the dielectric constant of the substrate is 0.635 mm and 9.6, respectively, and the height of the shielding box is 5.461 mm.

First, Table I shows the scalar potential calculation results at 5 GHz. For each source point, the data on the table were obtained by averaging over 25 different observation points evenly distributed on the substrate surface. The calculation by the method in [2] has been used as a reference value in calculating the error in the table. It typically needed around 1000 or more terms to get a convergent result with this method. The GPOF algorithm used five exponentials for the approximation in (2). On average, about 76 terms are involved in the calculation with the resultant error less than 0.166%.

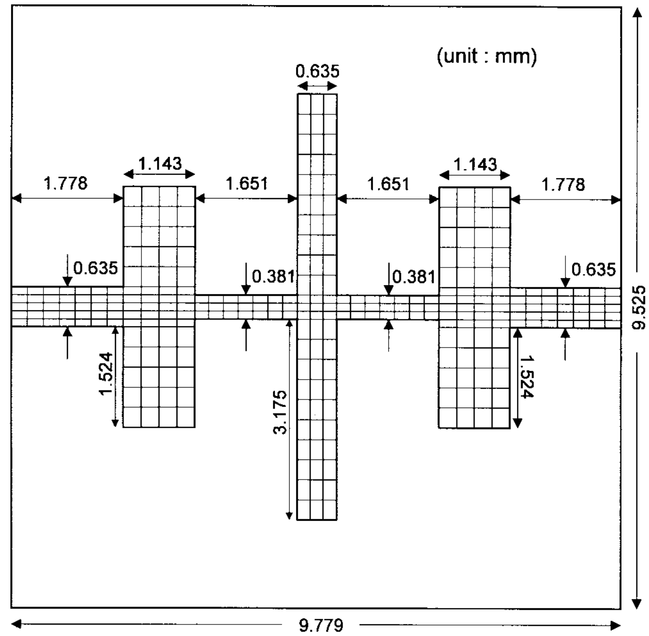

Fig. 1. Shielded microstrip low-pass filter.

TABLE I

(a) Average Number of Terms Required in the Calculation and (b) the Resultant % Error in the Scalar Potential (G^V) Calculation

x', y'	0.05a	0.15a	0.25a	0.35a	0.45a
0.05b	79.7	78.4	77.4	77.6	77.5
0.15b	78.7	77.4	75.6	76.5	75.8
0.25b	77.4	75.8	74.6	74.4	74.5
0.35b	77.6	76.9	74.0	74.2	74.0
0.45b	77.4	76.6	74.6	74.3	73.4

(a)

x', y'	0.05a	0.15a	0.25a	0.35a	0.45a
0.05b	0.42	0.18	0.15	0.28	0.27
0.15b	0.09	0.13	0.06	0.07	0.06
0.25b	0.11	0.07	0.24	0.11	0.13
0.35b	0.15	1.07	0.05	0.05	0.05
0.45b	0.11	0.06	0.08	0.12	0.04

(b)

A similar calculation for the $\hat{x}\hat{x}$ component of the vector potential Green's function required 62 terms on average with relative error 0.076% when four exponentials are used in the approximation.

For the next numerical example, the microstrip lowpass filter in Fig. 1 has been analyzed under the MPIE formulation with Galerkin MoM method using rooftop basis functions. In the MoM matrix element calculation, the coefficients of G_{i1} series has been precomputed since it does not change with position, and for G_{i2} series, only the first nine terms have been used in the computation, which corresponds to the original source and its eight nearest images weighted with the complementary error function. The present analysis used meshing scheme as shown in Fig. 1 with total 307 discretizations (536 unknowns). The average calculation time is 41.9 s per frequency on Sun Ultrasparc1 workstation. Fig. 2

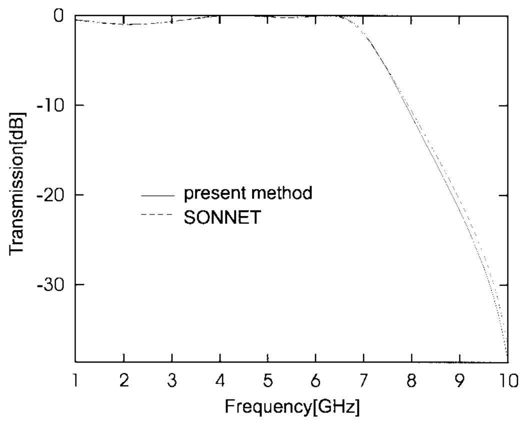

Fig. 2. Transmission characteristic of the filter.

shows the calculated transmission(S21) characteristic of the filter. The present calculation agrees quite well with the result by the commercial EM software SONNET using 1005 uniform discretizations.

IV. Conclusion

This letter has proposed an efficient method of summing the Green's function series in the shielded planar structures. A simple and efficient method has been used to transform the Green's function into the static image series, and the highly efficient Ewald method has been applied to the image series to yield an extremely fast convergent series. This technique is very efficient and accurate and therefore, will be a valuable tool in the efficient IE-MoM analysis of shielded planar circuits using arbitrary discretization scheme/basis functions.

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